Adaptive Multigrid Hurricane Modeling

Scott R. Fulton, PI
Department of Mathematics and Computer Science
Clarkson University, Potsdam, NY 13699-5815
315-268-2379 (voice) 315-268-2371 (FAX)
fulton@clarkson.edu

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http://www.clarkson.edu/~fulton

LONG-TERM GOALS

Accurate simulation and prediction of atmospheric and oceanic phenomena and circulation in general--and tropical cyclones in particular--relies on combining accurate data, detailed understanding of the relevant physics and dynamics, and appropriate numerical methods. This research project seeks to contribute to the latter two areas by:

- Developing improved numerical methods for modeling and predicting mesoscale phenomena
- Advancing understanding of tropical cyclone evolution and motion

Most of the work is focused on developing and applying adaptive multigrid methods in the context of a barotropic hurricane model. The techniques developed should be applicable to a wide range of modeling and predictive efforts, especially those requiring increased resolution in localized regions.

OBJECTIVES

Adaptive multigrid techniques offer the potential of fast computation and high accuracy; they differ from conventional nested grid methods by making full use of the interplay between discretizations on the different grids. Their potential has been demonstrated in a multigrid barotropic hurricane track model (MUDBAR) developed by the PI, which runs ten to twenty times faster than a uniform-resolution version for the same level of accuracy. A preliminary version of this model is described in Fulton (1997). Using this model as a base, our current research effort centers on the development of adaptive multigrid methods and their use in studying hurricane dynamics and motion. The principal objectives of this work are:

1. Development of self-adaptive multigrid techniques. The original version of the model used grids of fixed sizes which were simply moved to follow the storm. Our objective is to develop a fully self-adaptive model, using truncation error estimates generated during multigrid processing to decide automatically when and where to refine (or coarsen) the grids, thus approaching maximum accuracy and efficiency with minimum human intervention.

2. Implementation of higher-order accuracy. While the original version of the model used second-order conservative finite differences, higher-order methods have the potential to improve accuracy and efficiency. Extrapolation techniques (based on truncation error estimates computed with little additional work during multigrid processing) allow one to "bootstrap" from second-order to fourth-

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order accuracy while maintaining conservation properties. Consequently, our objective is to implement such a method and verify the accuracy and efficiency of the resulting model.

3. Extension of adaptive multigrid techniques to the shallow-water equations. The nondivergent barotropic model provides a good test bed for numerical techniques, but accurate prediction of tropical cyclone tracks requires improved dynamics. Our objective here is to extend the MUDBAR model to use the shallow-water equations; this will involve careful treatment of the grid interfaces and extending the higher-order multigrid techniques.

4. Sensitivity of hurricane tracks to vortex structure and grid resolution. Preliminary experiments have shown that small-scale changes in vortex structure may lead to significant differences in hurricane tracks. Our objective is to quantify the extent of these differences, and to compare them to those due to inadequate model resolution.

5. Studies of potential vorticity mixing and hurricane dynamics. Numerical simulations of potential vorticity mixing in hurricanes have been reported (Schubert et al, 1998); these were made with a doubly-periodic spectral model. It is possible that the preferential growth of wavenumber four asymmetries seen in that study may be influenced by the artificial assumption of periodicity. The MUDBAR model will allow the outer boundaries to be placed far away from the storm, and yet obtain sufficient resolution near the center to track the details of fine-scale mixing. Consequently, our objective is to repeat these numerical experiments to remove the possible effects of periodicity. In addition, we are continuing theoretical studies of equilibrium vortices (via the minimum enstrophy and maximum entropy principles), in an attempt to better understand the fundamental potential vorticity mixing process which underlies the dynamics of a hurricane.

APPROACH

This work involves both numerical and analytical approaches. The main focus is on developing and implementing new numerical methods; this requires analysis of the methods, implementation in the MUDBAR model, and extensive numerical experiments to quantify the model performance and verify the results of the analysis. Personnel involved in this effort include the PI and two graduate students (Nicole Burgess and Brittany Mitchell), both of whom joined this project as first-year M.S. students in June 1998. In addition to introducing these students to the research, the PI has focused on developing and coding the self-adaptive version of the model (objective 1 above) and identifying and verifying the proper way to compute truncation error estimates and perform extrapolation to higher-order accuracy. Nicole's main task (M.S. thesis project) is to implement higher-order accuracy in the model (objective 2 above); this work is just getting started. Likewise, Brittany's main task (M.S. thesis project) is to extend the model to the shallow-water equations (objective 3 above); this work is just getting started.

Studies of potential vorticity mixing in hurricanes (objective 5 above) also involve both analytical and numerical approaches. The PI has been working with Wayne Schubert at Colorado State University to develop analytical models of potential vorticity mixing in hurricanes via the minimum enstrophy and maximum entropy approaches; Mike Montgomery (also at Colorado State University) has been involved with this work. Professor Schubert's main contribution has been to identify the problem and guide the analytical solution, particularly for the nondivergent barotropic case, while the PI has worked on the details of the analytical formulation and numerical solution, particularly for the shallow-water case. In addition to the numerical evaluation of these analytical results, time-dependent numerical simulations using the MUDBAR model are also planned as noted above.

WORK COMPLETED

The key to both self-adaptivity and extrapolation to higher-order accuracy is the computation of accurate truncation error estimates. We have:

- Identified the proper grid transfer operators needed for truncation error estimates,
- Proved (analytically) the accuracy of such estimates and subsequent extrapolation,
- Verified these results computationally for the vorticity/streamfunction (Poisson) equation.

In particular, it turns out to be critical to transfer the forcing from the fine grid to the coarse grid using injection (rather than using the residual transfer operator, which is normally full weighting); without this, the accuracy of the truncation error estimate breaks down and extrapolation to higher order is not possible. This point is not apparent in published literature on the subject, e.g., Brandt (1977), Schaffer (1984). Building on this preparatory work, we have completed the following aspects of the main objectives:

1. Development of self-adaptive multigrid techniques. Coding of the self-adaptive multigrid scheme (including truncation error estimates, memory management, and adaptive grid refinement) is complete, and initial tests have begun. Results from this testing will be presented at the 23rd AMS Hurricane Conference on Hurricanes and Tropical Meteorology (Dallas, January 1999). A paper describing the self-adaptive model and quantifying its performance is in preparation.

2. Implementation of higher-order accuracy. While truncation error estimates have been implemented in the model for the vorticity/streamfunction equation, the extrapolation code has not yet been introduced.

3. Extension of adaptive multigrid techniques to the shallow-water equations. Work on this task has not yet begun.

4. Sensitivity of hurricane tracks to vortex structure and grid resolution. Preliminary experiments have been performed. Further work on this task will await the completion of the self-adaptive version of the model.

5. Studies of potential vorticity mixing and hurricane dynamics. For the nondivergent barotropic case, the analytical formulation of the minimum enstrophy problem has been extended to include inner and outer mixing radii and both momentum and energy constraints. The computer code for solving the resulting nonlinear system is about 80% complete at this time. For the shallow-water case, the minimum enstrophy problem has been properly formulated and the resulting variational problem solved analytically. The resulting Euler-Lagrange equations constitute a boundary value problem with four coupled nonlinear differential equations; a computer code for solving this problem is under construction. Results will be presented at the 23rd AMS Hurricane Conference on Hurricanes and Tropical Meteorology (Dallas, January 1999). A paper detailing the formulation and solution of the minimum enstrophy problems is in preparation.

In addition, the graduate students (new to this work in June 1998) have worked hard to develop the background needed to complete their tasks during the next year, and the computer equipment provided by the related DURIP grant has been installed.

RESULTS

The principal technical results achieved through this work in FY98 are:

- Rigorous analysis of the accuracy of truncation error estimates generated during multigrid processing. This analysis identifies the grid transfers needed to achieve high accuracy and thus will guide implementation of self-adaptive methods and extrapolation to higher order.

- Analytical formulation and solution of the minimum enstrophy problem for shallow-water vortices. This allows the prediction of the symmetric final state of an unstable hurricane-like vortex without having to solve for the complex nonlinear transient flow.

In addition, the self-adaptive version of the hurricane model is nearly complete, which will make possible the work planned on the other scientific objectives of this research during the next year.

IMPACT/APPLICATIONS

The self-adaptive multigrid hurricane model (MUDBAR) being developed should be fast and accurate. With appropriate data input, it could be utilized in operational forecasting of hurricane motion; in view of its speed and relative simplicity, it could be particularly appropriate for ensemble forecasting. While accurate prediction of hurricane tracks depends on many factors, this model should effectively eliminate the numerical method as a source of significant error.

The adaptive multigrid techniques studied here are quite general, and could be applied to many other modeling and predictive efforts requiring increased resolution in local regions, e.g., mesoscale and coastal zone models. Their principal benefit is an improved tradeoff between accuracy and efficiency, allowing either higher accuracy for the same amount of computer time or faster execution for the same level of accuracy.

The theoretical and numerical studies of potential vorticity mixing conducted in this project should lead to a deeper understanding of this fundamental process in the dynamics of hurricanes. This may impact our understanding (and eventual prediction) of hurricane intensity change and the development and evolution of small-scale destructive features such as mesoscale vortices.

TRANSITIONS

As the self-adaptive model is still under development, it has not yet been used by others researchers; any opportunities for collaboration in model testing, implementation, or intercomparisons would be welcomed. Numerical techniques being developed for solving the minimum enstrophy problem for hurricane-like vortices will likely be applied by members of Wayne Schubert's group for minimum enstrophy studies on the sphere.

RELATED PROJECTS

The PI received a DURIP grant (N00014-98-1-0368) which has provided funds for computer equipment used in this research.

A group at Colorado State University headed by Wayne Schubert and Mike Montgomery has been studying the potential vorticity mixing process. Work on this subject reported here has been done in close collaboration with this group; the PI has spent one month in Colorado each summer for the past few years working directly with this group.

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